

# Chiral Perturbation Theory, Large- $N_c$ and the $\eta'$ Mass

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## Abstract

Using chiral perturbation theory and the large- $N_c$  expansion, we obtain expressions for the  $\eta'$  mass and  $\eta - \eta'$  mixing in terms of low-energy chiral Lagrangian parameters. This is accomplished through an intermediate step of ‘matching’ the topological susceptibility in the large- $N_c$  and chiral Lagrangian descriptions. By inserting the values of well-measured parameters we obtain predictions involving the the second order parameters  $L_6, L_7$  and  $L_8$ . The prediction for  $L_6$  is quite restrictive even after allowing for  $1/N_c$  corrections.

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# 1 Introduction

Chiral perturbation theory (CHPT) provides a useful description of low-energy QCD in terms of the relevant light degrees of freedom [1]. These light degrees of freedom are simply the  $N_f^2 - 1$  pseudo-Goldstone bosons (PGB's) which result from the breaking of chiral symmetry:

$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V .$$

We do not include the  $U(1)_A$  symmetry in the above as it is violated by the axial anomaly [2]. The axial anomaly has physical effects due to the presence in QCD of field configurations with non-trivial winding number  $\nu$ . Were this not the case the chiral Lagrangian would have to be modified to accomodate an additional (ninth) pseudo-Goldstone boson, which has the quantum numbers of the  $\eta'$ . It is well-known that this is what occurs in the large  $N_c$  limit – the physical effects of the anomaly are suppressed [3] <sup>§</sup>. In the large- $N_c$  limit the low-energy effective Lagrangian contains the  $\eta'$  as well as the pions [5].

It might be regarded as a failure of the large- $N_c$  expansion that the  $\eta'$  is so heavy –  $m_{\eta'} = 958 \text{ MeV}$  – relative to the other strange PGB's such as the  $K^\pm, K^0, \bar{K}^0$  and  $\eta$ . However, we will argue here that this is a consequence of the smallness of the light quark masses and need not imply a breakdown in large- $N_c$  thinking. The point is that if one formally takes the limit  $N_c \rightarrow \infty$  with  $m_{u,d,s}$  fixed, it is easy to see that the  $\eta'$  becomes roughly degenerate with the strange PGB's (see section 2). However, it is more appropriate for the real-world to consider a double limit in which  $N_c \rightarrow \infty$  and  $m_{u,d,s} \rightarrow 0$  simultaneously, in some fixed ratio. Indeed, the breaking of the chiral  $SU(3) \times SU(3)$  symmetry associated with the strange quark mass  $\sim m_s/4\pi f_\pi \sim 15\%$  is still smaller than  $1/N_c$  ( $N_c=3$ )  $\sim 30\%$ . As we will discuss below, after taking this into account it becomes clear that the  $\eta'$  is heavy relative to the PGB's because of the smallness of  $m_s$  rather than a failure of large- $N_c$ . Using a simultaneous scaling of  $N_c$  and  $m_q$  we can contemplate a large- $N_c$  limit with the following hierarchy of masses:

$$m_{\rho, \text{baryon}} \gg m_{\eta'} \gg m_{\pi, K, \eta} . \quad (1)$$

We note that a similar point of view was taken by Leutwyler in [6], where a bound on the light quark mass ratios was derived using a double expansion in large- $N_c$  and the quark masses.

We will take the viewpoint that large- $N_c$  gives a qualitatively correct picture of QCD dynamics, and exploit the tension between the standard CHPT effective Lagrangian and the one resulting from large- $N_c$ . In particular, we will utilize the special status of the  $\eta'$  in

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<sup>§</sup>Despite this, the  $\theta$ -angle may still have physical consequences in the large- $N$  limit [3]. Also see [4] for a discussion of the possible large- $N_c$  behavior of instantons.

determining a quantity known as the topological susceptibility:

$$\frac{\langle \nu^2 \rangle}{V} = - \frac{1}{VZ} \left( \frac{\partial^2 Z}{\partial \theta^2} \right)_{\theta=0}, \quad (2)$$

where  $Z$  is the QCD partition function,  $V$  the volume of spacetime and  $\nu$  is the winding number given by

$$\nu = \frac{1}{32\pi^2} \int_V d^4x \, G\tilde{G}. \quad (3)$$

In the large volume limit,  $Z$  can be expressed in terms of the vacuum energy density  $\epsilon_0$ :

$$Z = \exp(-V\epsilon_0). \quad (4)$$

The problem is thus reduced to that of computing the  $\theta$  dependence of  $\epsilon_0$ . This can be done either in CHPT, which is equivalent to an expansion in the light quark masses  $m_q$ , or in the large- $N_c$  effective Lagrangian, which involves a double expansion in  $1/N_c$  and  $m_q$ . Equating the two results for the susceptibility will yield a relationship at leading order in  $N_c$  between  $m_{\eta'}$  and low-energy parameters appearing in CHPT. An alternative way of viewing our calculation is in terms of two different effective Lagrangians valid at the two energy scales  $\mu_1 \sim m_{\eta'}$  and  $\mu_2 \sim m_{\pi,K,\eta}$ . At the lower scale the  $\eta'$  has been ‘integrated out’. Requiring that both Lagrangians (one with the  $\eta'$ , the other without) yield the same susceptibility implies a non-trivial relation between their parameters.

In the following sections we will compute the topological susceptibility in both CHPT and Large- $N_c$  CHPT to obtain the desired relation. In the final section we will present our results and some final comments.

## 2 Large- $N_c$ Effective Lagrangian

As previously mentioned, at large- $N_c$  the effective Lagrangian must be modified to include an additional light meson, the  $\eta'$ , which plays the role of an additional PGB associated with  $U(1)_A$  [3, 5]. It is convenient to describe all  $N_f^2$  PGBs in terms of a  $U(N_f)$  matrix field  $U(x)$ . The  $\eta'$  is related to the phase of the determinant of  $U(x)$ :

$$\det U(x) = e^{-i\phi(x)} \quad (5)$$

with

$$\phi(x) = -\frac{\sqrt{2N_f}}{F} \eta'. \quad (6)$$

The effective Lagrangian at large  $N_c$  is

$$\mathcal{L}_N = \frac{F^2}{4} \text{tr}(\partial_\mu U^\dagger \partial^\mu U) + \Sigma \text{Retr}(MU^\dagger) - \frac{\tau}{2}(\phi - \theta)^2, \quad (7)$$

where the pion decay constant  $F \sim \mathcal{O}(N_c^{1/2})$ , the chiral symmetry breaking scale  $\Sigma \sim \mathcal{O}(N_c)$  (related to the quark condensate in the chiral limit) and  $\tau \sim \mathcal{O}(1)$  (related to the topological susceptibility of pure gluodynamics [3]). Note that the  $\theta$  angle has been absorbed from the mass matrix  $M$  into a shift of  $\phi$ .  $\mathcal{L}_N$  represents the leading order terms in the effective Lagrangian in the formal counting scheme where one takes the quark masses  $m_q$  in the mass matrix  $M$  to be  $\sim \mathcal{O}(N_c^{-1})$  and derivatives  $\partial \sim \mathcal{O}(N_c^{-1/2}) \sim m_\pi$ .

Taking the quark mass matrix to be (we now restrict ourselves to  $N_f = 3$  and the  $SU(2)$  isospin limit  $m_u = m_d = m$ )

$$\begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m_s \end{pmatrix} \quad (8)$$

we obtain the following mass relations:

$$\begin{aligned} m_\pi^2 &= \frac{2\Sigma m}{F^2} \\ m_\eta^2 &= \frac{2\Sigma(m + 2m_s)}{3F^2} \\ m_{\eta'}^2 &= \frac{2\Sigma(2m + m_s)}{3F^2} + \frac{6\tau}{F^2} \\ m_{\eta\eta'}^2 &= \frac{2\sqrt{2}\Sigma(m - m_s)}{3F^2} . \end{aligned} \quad (9)$$

The first two equations are identical to the results of CHPT, and the last equation describes  $\eta - \eta'$  mixing. Note that both the anomalous and CHPT contributions to  $m_{\eta'}^2$  are formally  $\mathcal{O}(1/N_c)$  in  $N_c$  counting. As mentioned previously it is possible to take the quark masses  $m_q \sim \mathcal{O}(N_c^{-1}) \rightarrow 0$  as  $N_c \rightarrow \infty$  such that the anomalous term continues to dominate – i.e.,  $\tau \gg \Sigma m_q$ . This is the limit that we will consider. In real QCD where the  $N_c \rightarrow \infty$  limit is only approximate the anomalous contribution to  $m_{\eta'}$  is presumably not suppressed below the CHPT contribution which is small because of the quark masses. We argue therefore that the large  $N_c$  form of (7) is appropriate to QCD.

The parameter  $\tau$  can be extracted at leading order by requiring that the eigenvalues of the  $\eta - \eta'$  mass matrix correspond to the phenomenologically observed values of 547 and 958 MeV respectively. If we allow CHPT and  $1/N_c$  uncertainty of 50% in the mass relations (9) (from, e.g., uncertainties in  $F_{\eta'}$ , and subleading interactions not in (7) which shift the mass relations (9) etc.), we obtain the result

$$(137 \text{ MeV})^4 < \tau < (202 \text{ MeV})^4 . \quad (10)$$

In our calculations, we have used  $F = 93 \text{ MeV}$ , the allowed range of light quark masses  $m_u \simeq m_d \simeq 7 \text{ MeV}$  and measured pion and kaon masses to conclude that  $m_s \simeq 150 \text{ MeV}$  and  $\Sigma = \mathcal{O}(200 \text{ MeV})^3$ .

Note that the above results are consistent with the claim that  $\tau \geq \Sigma m_s$  mainly due to the smallness of  $m_s$ .

The next step is to compute the topological susceptibility using  $\mathcal{L}_N$ . This requires minimizing the potential energy at small  $\theta$  angle and differentiating twice with respect to  $i\theta$ . The  $SU(N_f)$  isospin limit ( $m = m_s$ ) of this problem was analyzed previously by Leutwyler and Smilga [7]. In this case the global minimum of the potential occurs when  $U$  is a multiple of the unit matrix  $U = e^{-i\phi/N_f}$ . The vacuum energy is of the form

$$\epsilon_0 = -N_f \Sigma m + \frac{1}{2} \theta^2 \frac{\tau \Sigma m}{N_f \tau + \Sigma m} + \mathcal{O}(\theta^4), \quad (11)$$

and the topological susceptibility at  $\theta = 0$  is

$$\frac{\langle \nu^2 \rangle}{V} = \frac{\tau \Sigma m}{N_f \tau + \Sigma m}. \quad (12)$$

In the limit we are considering, the denominator is dominated by  $\tau$ , so

$$\frac{\langle \nu^2 \rangle}{V} \simeq \frac{\Sigma m}{N_f} \left( 1 - \frac{\Sigma m}{N_f \tau} \right). \quad (13)$$

The second term is  $\mathcal{O}(m^2)$ , and one may wonder whether it is consistent to retain it given the neglect of terms of higher order in  $(\Sigma MU)$  in  $\mathcal{L}_N$ . The point is that due to an additional power of  $\Sigma$  the term in (13) is larger by a factor of  $N_c$  than the subleading corrections we have neglected. When we match topological susceptibilities with the true low-energy theory described by the  $SU(N_f)$  chiral Lagrangian (no  $\eta'$ ), this term will fix a combination of  $L_{6,7,8}$  at leading order in  $N_c$ .

It is important to note that the  $\eta'$  and the PGB's play a unique role in determining  $\frac{\langle \nu^2 \rangle}{V}$ . The other particles not described by  $U(x)$ , such as the  $\rho$  meson, or the baryons  $p, n, \dots$  are expected to have zero expectation value in the vacuum, and hence do not contribute to the susceptibility even if their interactions have some  $\theta$ -dependence. This observation is independent of large- $N_c$ , which only guarantees that (12) is accurate to leading order.

The extension of (12) to the case of a heavy strange quark is straightforward, and we obtain

$$\frac{\langle \nu^2 \rangle}{V} = \frac{\tau \Sigma \mathcal{M}}{\tau + \Sigma \mathcal{M}}, \quad (14)$$

where  $\mathcal{M} = (1/m_u + 1/m_d + 1/m_s)^{-1} = mm_s/(m + 2m_s)$ . Note that this reduces to  $m/N_f$  where  $N_f = 3, 2$  for  $m_s \rightarrow m$  and  $m_s \rightarrow \infty$  respectively.  $\Sigma \mathcal{M}$  is approximately  $\Sigma m$ , which is determined by the pion mass relation in (9).

The range in  $\tau$  given in (10) translates to the following range in the topological susceptibility

$$3.7 \times 10^7 < \frac{\langle \nu^2 \rangle}{V} < 4.05 \times 10^7 \quad (15)$$

The uncertainty in  $\tau$  does not lead to a large variation in  $\langle \nu^2 \rangle / V$ . There are, however, additional  $1/N_c$  corrections to the relation (14) which we have not yet examined in detail. As mentioned previously, the uncertainties represented in (15) are due to  $1/N_c$  corrections to the mass relations (9). However, subleading interactions which do not appear in (7) may also alter the calculation of the susceptibility.

There are two types of subleading corrections, both formally of order  $1/N_c$ , which we must consider.

- Higher orders in  $(\Sigma MU)$  which induce corrections suppressed by  $\mathcal{O}(m_q)$ . The most important of these are of the form of the  $L_{6,7,8}$  interactions in (19) below. In large- $N_c$  counting  $L_{6,7}$  are suppressed by Zweig's rule so it is  $L_8$  which is most important ¶.
- $\mathcal{O}(1/N_c)$  interactions of the  $\eta'$  in  $\mathcal{L}_N$  which violate the  $U(N_f) \times U(N_f)$  flavor symmetries but which preserve  $SU(N_f)$  symmetries. These corrections represent the fact that the  $\eta'$  is only a PGB in the large- $N_c$  limit. The most important of these is of the form

$$(\phi - \theta)^2 \Sigma \text{Retr}(M e^{i\theta/N_f} U^\dagger) , \quad (16)$$

where the coefficient is  $\mathcal{O}(1/N_c)$ .

We can estimate the size of the uncertainties introduced in the large- $N_c$  susceptibility by corrections of the above type. Naively, the  $L_{6,7,8}$  interactions will lead to a correction of order  $\mathcal{O}(m_s)$  which would be of order 15%. However, the issue is somewhat more complicated than this. The structure of  $L_8$  is such that its effect on the calculation of the susceptibility is equivalent to a shift in the quark masses in (14):  $m_i \rightarrow m_i(1 + \mathcal{O}(m_i))$ . Hence only  $m_s$  receives a non-negligible correction. However, the strange quark mass has only a very small effect on the susceptibility because it is so much more massive than the u,d quarks. The parameter  $\mathcal{M} = m(1 + \mathcal{O}(m/m_s))$  in (14) depends only weakly on  $m_s$ . The  $L_{6,7}$  operators, although suppressed relative to  $L_8$  by a power of  $1/N_c$ , have a larger effect on the susceptibility. An explicit calculation shows that  $L_{6,7}$  shift the susceptibility by  $\mathcal{O}(m_s 1/N_c)$ , leading to a larger uncertainty than is given by the range (15). This is the dominant subleading correction. We will associate an additional error of  $\sim 5\%$  to this correction, leading to the following range of susceptibility (representing roughly 10% error) :

$$3.5 \times 10^7 < \frac{\langle \nu^2 \rangle}{V} < 4.3 \times 10^7 . \quad (17)$$

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¶In the real world the central values of  $L_{6,7}$  are roughly  $\sim (1/N_c)L_8$ , which is at least consistent with the  $1/N_c$  expansion approximation.

The second type of correction leads to a only a small uncertainty in the susceptibility. Let  $\phi_0$  be the value of  $\phi$  which minimizes the vacuum energy  $\epsilon_0$  in (11) at non-zero  $\theta$ :

$$\phi_0 = - \frac{\theta}{(1 + \Sigma\mathcal{M}/\tau)} . \quad (18)$$

We can then estimate the shift in the susceptibility by substituting  $\phi_0$  into (16) and differentiating twice with respect to  $\theta$ . This leads to a correction which is suppressed by a factor of  $\mathcal{O}(N_c^{-1} \Sigma\mathcal{M}/\tau)$ , making it less than 5% of (14).

### 3 Chiral Lagrangian

We next compute the topological susceptibility using the standard  $SU(N_f)$  chiral Lagrangian appropriate below the  $\eta'$  mass. The heavy  $\eta'$  has been ‘integrated-out’ and its effects are exhibited only in the chiral coefficients. The susceptibility only depends on the potential of the chiral Lagrangian. Terms with derivatives are irrelevant.

At  $\mathcal{O}(m^2)$  the potential is [1]

$$\begin{aligned} \mathcal{V}_\chi = & - \Sigma \text{Retr}(M_\theta U^\dagger) - L_6 \frac{16\Sigma^2}{F^4} (\text{Retr} M_\theta U^\dagger)^2 - L_7 \frac{16\Sigma^2}{F^4} (\text{Imtr} M_\theta U^\dagger)^2 \\ & - L_8 \frac{8\Sigma^2}{F^4} \text{Retr}(M_\theta U^\dagger M_\theta U^\dagger) , \end{aligned} \quad (19)$$

where the mass matrix  $M_\theta$  now incorporates the  $\theta$ -angle,  $M_\theta = M \exp i\theta/N_f$ . Here the  $U(x)$  matrix field is restricted to unit determinant and is parameterized as

$$U(x) = \exp(i2\pi^a(x)T^a/F) . \quad (20)$$

The topological susceptibility is again found by minimizing the potential for  $U$  and taking the second derivative of  $Z$  with respect to  $(i\theta)$  at the minimum. We find

$$\begin{aligned} \frac{\langle \nu^2 \rangle}{V} = & \frac{2}{9} \left[ \frac{\Sigma(2m + m_s)}{2} + \frac{16\Sigma^2}{F^4} ( (L_6 + L_7)(2m + m_s)^2 + L_8(2m^2 + m_s^2) ) - \right. \\ & \left. \frac{4(m_s - m)^2 ( \frac{\Sigma}{2} + \frac{16\Sigma^2}{F^4} ( (L_6 + L_7)(2m + m_s) + L_8(m + m_s) ) )^2}{\Sigma(m + 2m_s) + \frac{16\Sigma^2}{F^4} (2L_6(2m + m_s)(m + 2m_s) + 4L_7(m_s - m)^2 + 2L_8(m^2 + 2m_s^2))} \right] . \end{aligned} \quad (21)$$

By equating this result, which is a function of the second order parameters of the chiral Lagrangian (19), with the range (15) determined from the large  $N_c$  expansion we can impose constraints on the values of the parameters  $L_6$ ,  $L_7$  and  $L_8$  in the chiral Lagrangian (19).

The expression (21) has some interesting properties. The leading order behavior is

$$\frac{\langle \nu^2 \rangle}{V} = \Sigma\mathcal{M} + \dots , \quad (22)$$

which agrees with the leading order large- $N_c$  result (14) for  $\tau \gg \Sigma\mathcal{M}$ . Expanding (21) in powers of the light quark masses, we can see that its leading dependence on  $L_6$  is  $\sim L_6(\Sigma m)(\Sigma m_s/F^4)$ , whereas the leading dependence in  $L_{7,8}$  is  $\sim L_{7,8}(\Sigma m)(\Sigma m/F^4)$ . This implies that the susceptibility is much more sensitive to changes in  $L_6$  than  $L_{7,8}$ . When we perform the matching of susceptibilities below, we will find that  $L_6$  is more restricted than  $L_{7,8}$ .

We can derive further constraints on the  $L_{6,7,8}$  by matching higher derivatives of the vacuum energy with respect to  $i\theta$  in the large- $N_c$  and chiral descriptions. This corresponds to matching  $\langle\nu^4\rangle$ ,  $\langle\nu^6\rangle$ , etc. Unfortunately, we have checked that in the chiral Lagrangian description  $\langle\nu^4\rangle$  is still only weakly dependent on  $L_{7,8}$ . The leading dependence on  $L_6$  is again larger by a factor of  $\mathcal{O}(m_s/m)$  than the dependence on  $L_{7,8}$ . Thus we do not believe that additional constraints on  $L_{7,8}$  will be obtained by further matching.

## 4 Matching

A fit to the parameters  $L_6, L_7$  and  $L_8$  from low energy data yields the following one-sigma range of values [1]:

$$\begin{aligned} L_6 &= -.0002 \pm .0003 \\ L_7 &= -.0004 \pm .0002 \\ L_8 &= +.0009 \pm .0003 \quad . \end{aligned} \tag{23}$$

Upon matching the results for the topological susceptibility in the large  $N_c$  (14) and standard chiral lagrangian (21) we obtain a more restrictive prediction for the volume in  $L_6, L_7$  and  $L_8$  space consistent with the observed meson masses upto the errors described above in (17). The combined constraints are shown in Fig. 1. The parameter  $L_6$  is well determined by the fit.  $L_7$  and  $L_8$  are not so well constrained. In Fig 2 and Fig.3 we project our limits onto the  $L_6 - L_8$  and  $L_6 - L_7$  planes. Figure 2 shows the allowed regions of  $L_6 - L_8$  with  $L_7$  unconstrained and Figure 3 shows the allowed regions of  $L_6 - L_7$  with  $L_8$  unconstrained.



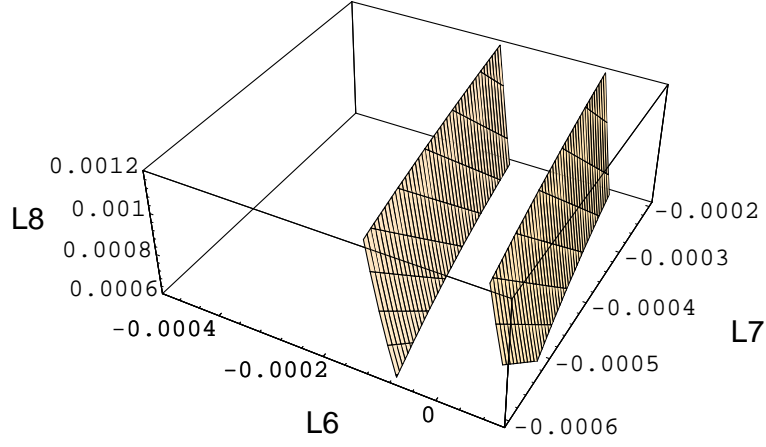


Fig.1: The region of  $L_{6,7,8}$  space consistent with the observed meson masses.

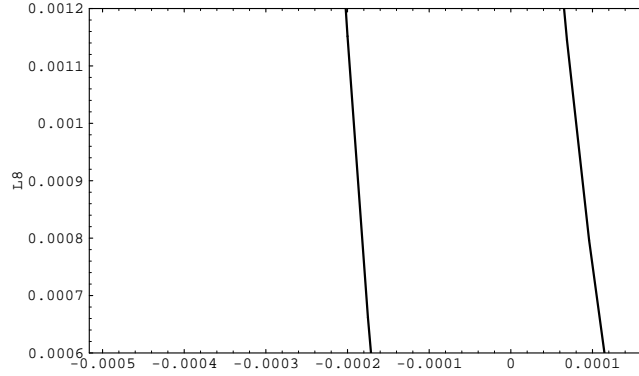


Fig.2: Projection of allowed region of parameter space onto  $L_6 - L_8$  plane.

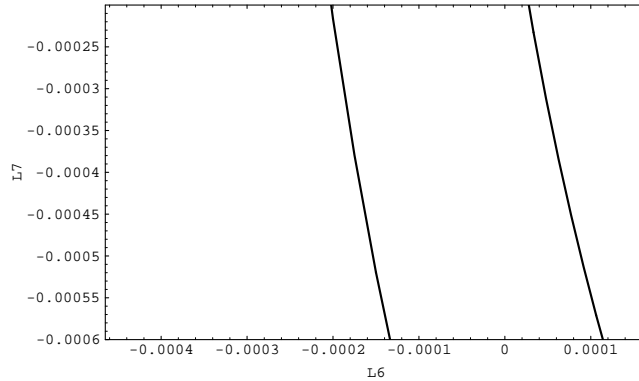


Fig.3: Projection of allowed region of parameter space onto  $L_6 - L_7$  plane.

## 5 Discussion

The  $\eta'$  has certain special properties in QCD as it is ‘almost’ the Goldstone boson associated with spontaneously broken  $U(1)_A$  symmetry. Within the large- $N_c$  expansion this ‘almost’ can be made quantitative and relations derived (see (14)) between the  $\eta$  and  $\eta'$  masses and the topological susceptibility. The latter quantity measures the sensitivity of the QCD vacuum energy to small changes in  $\theta$  at  $\theta = 0$ . We can also compute the susceptibility directly in chiral perturbation theory. The leading order result in CHPT is precisely that obtained from the large- $N_c$  Lagrangian if the  $\eta'$  is taken to be very massive ( $\tau \gg \Sigma\mathcal{M}$ ). However, the subleading corrections in CHPT come from  $L_{6,7,8}$  whereas in the large- $N_c$  expansion they depend on the ratio  $\Sigma\mathcal{M}/\tau$ . We have exploited this observation to derive what are essentially large- $N_c$  predictions for the second order CHPT coefficients (particularly  $L_6$ ) in terms of the  $\eta'$  mass which is taken as an input. Our prediction for  $L_6$  is restrictive even when  $1/N_c$  corrections are included. The central value we predict is within the  $1\sigma$  region previously extracted from low-energy data (also using large- $N_c$ ) [1].

A simple way of understanding our calculation is in terms of effective Lagrangians valid at two scales,  $\mu_1 \sim m_{\eta'}$  and  $\mu_2 \sim m_{\pi,K,\eta}$ . At the lower scale the  $\eta'$  has been ‘integrated out’. The effect of virtual  $\eta$ ’s appears in the coefficients  $L_{6,7,8}$  and is determined at leading order in large- $N_c$ . In this sense our calculation is similar to that of [8], which estimated the parameters  $L_i$  from resonance exchange.

## Acknowledgements

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